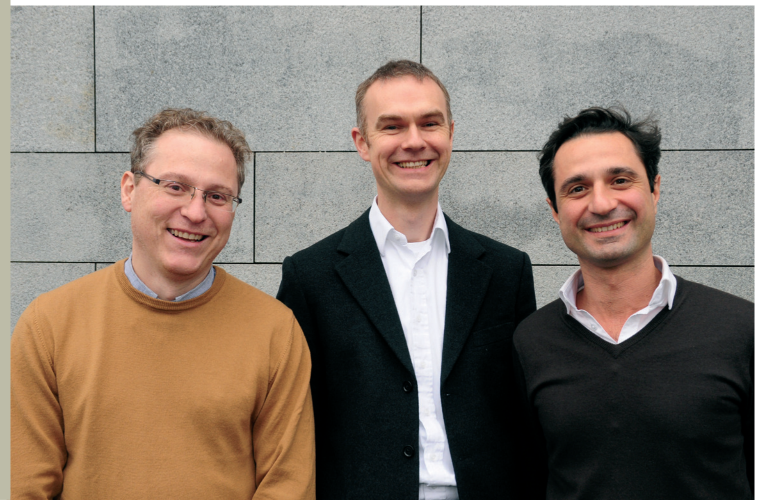
PEACE AND QUIET, TIME TO THINK



From left to right:
Carlo Gasbarri
Stefan Kebekus
Gianluca Pacienza
(Joint Research Group, Mathematics)

"In order to work, a mathematician needs peace and quiet to think," explains Prof Dr Stefan Kebekus. It is precisely these two fundamental resources that he hopes to gain by taking up a Senior Fellowship at FRIAS. The project developed by the Freiburg-based mathematician and his colleagues in Strasbourg, Prof Dr Carlo Gasbarri and Dr Gianluca Pacienza, is entitled "Rational Points, Rational Curves and Automorphisms of Special Varieties". Their collaboration is one of four cross-border fellowship groups, which are all supported jointly by FRIAS and the University of Strasbourg Institut d'Études Avancées (USIAS). It is also the first project from the pure mathematics field carried out at FRIAS.

The principal investigators, Kebekus and Pacienza, study algebraic geometry. The third, Gasbarri, arithmetic geometry. So, is it possible to explain the research undertaken in "Rational Points, Rational Curves and Automorphisms of Special Varieties" using metaphors from school maths?

Well, we can try: we remember from the topic of curve sketching that each equation correlates to an object that becomes visible within a system of coordinates. Mathematicians speak of these in a very abstract way as "spaces". Gasbarri, Kebekus and Pacienza are interested in those spaces that are given by particularly simple algebraic equations. So now comes the question: how many different spaces are there? Is it possible to differentiate between certain basic types? Is there a meaningful classification system? In the case of one-dimensional spaces, these questions were answered at the beginning of the 20th century.

But what answers do we find for higher dimensions? How many potential spaces are there? Can they be classed into groups, and what basic spaces are common to all others?

"When it comes to two-dimensional spaces," says Carlo Gasbarri, "they are fairly well understood by algebraic geometry – and are therefore not so interesting for Kebekus and Pacienza. But in my field there are still a few problems to work on." At three dimensions and above, both arithmetic and algebraic geometry have their work cut out for them.

"We want to gain an overview of these spaces," explains Stefan Kebekus. "To a certain extent we are being guided by Aristotle's idea of a system of all things."

However, the mathematicians are not satisfied with the three dimensions accessible through their intuition. "We are observing spaces that have a terrifying number of dimensions. And computers are no help here. They might work at computing really simple examples for weeks on end and then suddenly abort because the main memory is full."

Theoretical investigation is therefore the tool of choice. The researchers are looking for principles that will allow general statements to be made about the geometry and classification of n-dimensional spaces. They are conducting this search with pen and paper.

This was precisely the approach adopted by Bernhard Riemann in the 19th century, and then in the 20th by David Mumford, who built on Riemann's work. These two pioneers of a mathematical field that has seen dramatic growth, particularly in the last decade, proved that although there may be an infinite number of spaces, a finite number of basic types exist. "This made the problem one that could be tackled," says Kebekus, "because it means there is a system. Once that is found, it will be possible to make generalised statements."

Gasbarri describes the difference between his own research field and that of Kebekus as follows: "Algebraic geometry is interested in structures and systems of classification – the overall picture. For example, are there symmetries in the geometry of these spaces? Arithmetic geometry on the other hand is concerned with the solutions to an equation. Am I able to find points on the surface whose coordinates are whole numbers? Or even special cases such as Pythagorean triples?" In view of the complexity of the individual equations, arithmeticians such as Gasbarri also look for families of solutions that simultaneously answer the largest number of problems possible.

The equation $x^2+z^2 = y^3(1-y)^3$ of Citric appears as simple as the figure itself. Two cusps mirror-symmetrically arranged rotate around the traversing axis.
Source: Herwig Hauser, CC BY-NC-SA, http://imaginary.org/gallery/herwig-hauser-classic

Are the researchers already able to foresee applications for their work? "Experience to date shows that good maths has always found a use," states Kebekus. "Riemann could never have dreamt that his findings would play such a key role in coding technologies." It is no coincidence that the NSA, a major U.S. secret service, is currently the world's largest employer of mathematicians. There are also links to computer science, and many areas of crossover with theoretical physics. "Perhaps it will be another 150 years before the mathematical insights we gain today find their application."

Kebekus appreciates the fact that as a geometrist he moves between two different worlds. "The most exciting thing about our subject area is that there are two descriptions for everything. On the one hand there is the equation, which is investigated in algebra in a very structured, formal way. And on the other is a geometric space which I am able to capture using my intuition."

When both systematic approaches and intuition falter, the mathematician leaves his quiet office and seeks a colleague. "Mathematical insights are achieved through discussions with people, who ideally have an extremely broad education," Kebekus explains. And Gasbarri adds: "By trying to explain to a colleague why a question poses a particular problem, you find a solution – in an ideal world, of course. Unfortunately it's not often that easy."

"Our field is traditionally one that sees a lot of collaboration," says Kebekus. "If I notice that someone else is interested in my subject area, we get together." This is why the working environment at an institute for advanced study provides a format ideally suited to their science.

The FRIAS-USIAS "joint group" programme is intended to strengthen the collaboration between the two universities and at the same time enhance the international visibility of the upper Rhine valley as a research campus. The attractiveness in combining the expertise of the two universities becomes visible when you look at the list of eminent guest researchers who Kebekus and Gasbarri could attract to Freiburg and their project. "I am looking forward to the many opportunities for exchange and believe that the fellowship will enable us to establish a great number of contacts for exciting discussions," Gasbarri concludes. The strategic partnership between FRIAS and USIAS has started very successfully.
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